Quantum coherence generated by quantum interference

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This paper deals with a system exhibiting quantum interference in the two-photon coupling between its ground state and the continuum through two quasiresonant intermediate levels by two modes of the electromagnetic field. We show that this system can generate quantum coherence among the two field modes, the internal and the external degrees of freedom. Two applications are discussed: generation of photon-number correlations between the two modes, and selection of atoms in a given velocity class.

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I. INTRODUCTION

In this paper we shall study the possibility of generation of quantum coherence through selective two-photon ionization of an atom. Let us briefly review the main aspects of quantum coherence, correlation and entanglement.

Consider two quantum systems S_1 and S_2, with Hamiltonian operators H_1 and H_2 having sets $\{E_i(\alpha_i),|\psi_i(\alpha_i)\rangle\}$ ($i=1,2$) of eigenvalues and eigenvectors (α_i represents a set of quantum numbers that characterizes completely the state of the system i). If the two systems do not interact, quantum mechanics states that the global Hamiltonian is just the sum of the two individual Hamiltonians: $H=H_1+H_2$. The corresponding set of eigenvalues and eignevectors for the total system S_1+S_2 is $\{E_1(\alpha)+E_2(\alpha'),|\psi_1(\alpha)\rangle|\psi_2(\alpha')\rangle\}$. Otherwise stated, the base of eigenvectors is factorizable in terms of the eigenvectors of each individual system. This property implies the following corollaries: (1) The density matrix ρ of the global system in the base formed by the global eigenvectors is also diagonal. (2) The expectation value of a global observable $\hat{O}=\hat{O}_1\hat{O}_2$, where $\hat{O}_i$ acts only on the variables of the system S_i, is also factorizable: $\langle\hat{O}\rangle=\langle\hat{O}_1\rangle\langle\hat{O}_2\rangle$. These corollaries are promptly demonstrated by projecting onto the basis of eigenstates. Corollary (1) means that there is no coherence between the two systems, whereas corollary (2) means that the correlation function for the operator $\hat{O}$ vanishes. On the contrary, if there is a term in the Hamiltonian that couples the two systems, the factorization property for the global eigenvectors is not verified, and the general form of an eigenvector $|\Psi\rangle$ of the global system when expressed in the base formed by the products $|\psi_1(\alpha)\rangle|\psi_2(\alpha')\rangle$ is $|\Psi\rangle=\Sigma_{\alpha,\alpha'}A(\alpha,\alpha')|\psi_1(\alpha)\rangle|\psi_2\times(\alpha')\rangle$. The state $|\Psi\rangle$ is often said to be an "entangled" state in this base. Correspondingly, the density matrix has nonzero off-diagonal elements, reflecting the existence of coherence between the two systems and the operator $\hat{O}$ has a nonzero correlation function: $C_\Psi(\hat{O})=\langle\hat{O}_1\hat{O}_2\rangle-\langle\hat{O}_1\rangle\langle\hat{O}_2\rangle\neq 0$.

Although we have been considering here, for the sake of clarity, the case of two (interacting or not) systems, these ideas can be shown to apply to different degrees of freedom of a system: suppose that two degrees of freedom correspond to commuting observables $\hat{D}_1$ and $\hat{D}_2$. If the corresponding Hamiltonian is of the form $H=H_0+H_1(D_1)+H_2(D_2)$, in which H_0 does not depend on (and thus commutes with) $\hat{D}_1$ and $\hat{D}_2$, then the eigenvalues of the system will be of the form $E_0+E_1+E_2$, where E_i are eigenvalues of H_i, and the eigenvectors will be of the (factorized) form $|\psi_0\rangle|\psi_1\rangle|\psi_2\rangle$, with $|\psi_i\rangle$ being an eigenvector of H_i. Correspondingly, a coupling potential $V(D_1,D_2)$ will induce coherence between the two degrees of freedom. All the preceding results apply to the present case. The only restriction is that the degrees of freedom correspond to commuting observables

Consider now a coupling potential $V(t)$ [with $V(t)=0$ for $t<-T$ or $t>T$] between the two previously uncoupled systems. V is a coupling potential if it contains *products* of operators acting on the systems S_1 and S_2 (so that the global Hamiltonian is no more the sum of the Hamiltonians of each system) in which case the eigenvectors of the global system do not present the factorizable form. Then, even for $t>T$ (after the end of the interaction), the state describing the global system S_1+S_2 in general remains an entangled state: its representation in the base constituted by the tensorial product of the eigenstates of S_1 and S_2 is nonfactorizable. According to the discussion above, even after the end of the interaction, there will be quantum coherence between the two systems, and a global observable will in general have a nonzero correlation function. This property of quantum systems — the survival of coherence after the end of any interaction — has no classical analog and is at the origin of many intriguing problems, as, e.g., the famous Einstein-Podolsky-Rosen (EPR) paradox [1] and Bell inequalities [2]. The EPR paradox is a particularly beautiful example of the correspondence between correlation and entanglement.

From the point of view of the applications, second-order quantum coherence in one [3] or two [4,5] modes of the light field has allowed measurements with reduced quantum noise (squeezing), and the generation of coherence between internal and external atomic degrees of freedom by the interaction

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with laser light has been used in laser cooling and trapping of atoms to temperatures in the micro-Kelvin range [6–10].

The coherence properties of the light field are naturally affected by the interaction with atomic systems. In particular, it has been recently shown that an atomic system possessing a particular level structure can restore the quantum coherence destroyed by losses [11].

In this paper, we study a system consisting of an "atom" (i.e., a system possessing discrete and continuous parts in its energy spectrum) interacting with the electromagnetic field. The atom has two degrees of freedom (internal state and center-of-mass movement) and interacts with two modes of the electromagnetic field. The unperturbed Hamiltonians corresponding to these degrees of freedom do commute (in the dipole approximation), thus there is equivalence in talking about correlation, coherence or entanglement. Obviously, the precise type of entanglement, coherence or correlation, that will show up will depend on the details of the problem. In this context, we show how quantum coherence can be generated by quantum interference. The coherence may involve the various degrees of freedom of the system: internal states, external states (center-of-mass motion), and electromagnetic field states (photon number) can become correlated as the interference filters some combinations of these states. The aim of the present paper is to discuss the physical mechanisms underlying these phenomena.

The paper is organized as follows: in Sec. II we present our simple model, which will serve as a paradigm for the subsequent sections; we discuss the basic mechanism by which quantum interference can lead to the generation of quantum coherences; and we outline a realistic physical system having this property. In Sec. III we discuss the generation of coherence between the internal atomic state and the field modes thanks to the light shift. In Sec. IV we study the mechanism producing correlations between internal and external degrees of freedom due to Doppler effect, allowing velocity class selection. Finally, Sec. V summarizes the main results obtained in the paper.

II. QUANTUM INTERFERENCE GENERATING QUANTUM COHERENCE

Consider a system with the level structure shown in Fig. 1, that will be called "the atom." The levels are coupled by two modes of the electromagnetic field: Mode 1 couples the ground state $|g\rangle$ to the intermediate state $|e_1\rangle$ and the intermediate state $|e_2\rangle$ to the continuum. Mode 2 couples the ground state to the state $|e_2\rangle$ and the state $|e_1\rangle$ to the continuum. The two modes have the same frequency ω, and the intermediate states $|e_1\rangle$ and $|e_2\rangle$ have detunings resp. δ_1 and δ_2 with respect to ω. The continuum state $|s(\delta_s)\rangle$ has a detuning δ_s with respect to 2ω and the density of states is $f(\delta_s)$. The field modes are labeled by the photon numbers n_1 and n_2. The states of the total atom+field system will be noted $|a;n_1,n_2\rangle$, with $a=\{g,e_1,e_2,s(\delta_s)\}$.

The dynamics of the global system is described by a Hamiltonian H of the form

$$H=H_0+V \tag{2.1}$$

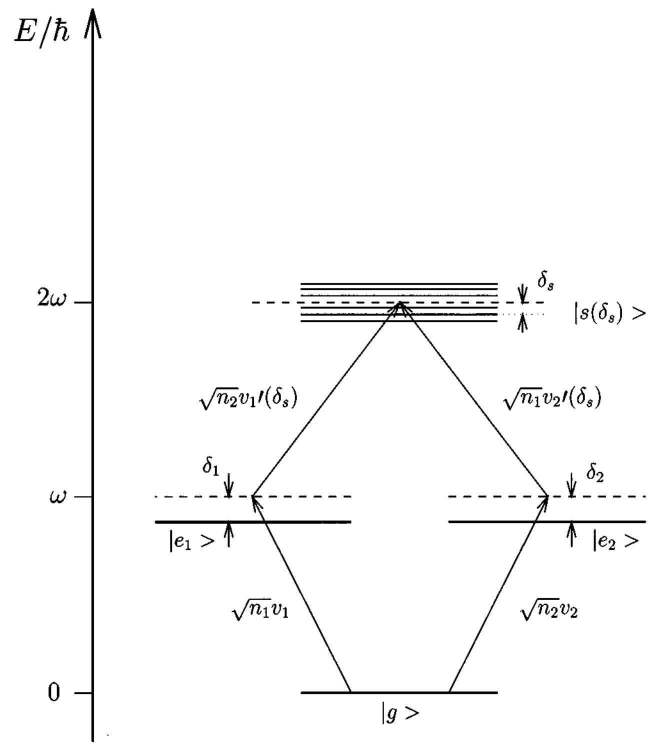

FIG. 1. Level structure and electromagnetic couplings for the "atom." Mode 1 couples the ground state to the intermediate state $|e_1\rangle$ and the intermediate state $|e_2\rangle$ to the continuum, whereas mode 2 couples the ground state to the intermediate state $|e_2\rangle$ and the intermediate state $|e_1\rangle$ to the continuum. We neglect the noninterfering couplings corresponding to the absorption of two photons in the same mode (cf. text).

with H_0 being the total atom+field unperturbed Hamiltonian and V the coupling perturbation, with nonzero matrix elements given by

$$\langle e_1;n_1-1,n_2|V|g;n_1,n_2\rangle=\sqrt{n_1}v_1, \tag{2.2a}$$

$$\langle e_2;n_1,n_2-1|V|g;n_1,n_2\rangle=\sqrt{n_2}v_2, \tag{2.2b}$$

$$\langle s(\delta_s);n_1-1,n_2-1|V|e_1;n_1-1,n_2\rangle=\sqrt{n_2}v_1'(\delta_s) \tag{2.2c}$$

$$\langle s(\delta_s);n_1-1,n_2-1|V|e_2;n_1,n_2-1\rangle=\sqrt{n_1}v_2'(\delta_s). \tag{2.2d}$$

Because of its two-photon coupling to the continuum the ground state $|g;n_1,n_2\rangle$ acquires a finite lifetime. From second order perturbation theory [12,13], one finds a transition rate (inverse lifetime) given by

$$\Gamma_g=\frac{2\pi}{\hbar^2}f_0n_1n_2\left|\frac{v_1v_1'}{\hbar\delta_1'}+\frac{v_2v_2'}{\hbar\delta_2'}\right|^2, \tag{2.3}$$

where we put $f_0=f(0)$, $v_1'=v_1'(0)$, $v_2'=v_2'(0)$.

In writing this equation, we neglected two other types of couplings connecting the ground state to the continuum, namely, those corresponding to the absorption of two photons of the same mode. These couplings are not subject to interference and are not interesting for the present discussion. This hypothesis will be discussed later in this section. For the moment, we keep the simple model in which the ionizing photons are absorbed from different modes and the corresponding amplitudes do interfere.

As is well known, the energy levels of an interacting atom are shifted by various perturbating effects. If these shifts are comparable to the laser-atom detunings δ_1 and δ_2 (measured in the absence of perturbations), the ionization process can become sensitive to these shifts. In Eq. (2.3) the detunings δ_1' and δ_2' include the laser-atom detuning and the perturbations. We take into account here two kinds of perturbations: the effects of the light shift (i.e., the energy shift due to the nonresonant interaction with the electromagnetic field) and of the Doppler effect, due to the motion of the atom:

$$\delta_1' = \delta_1 + \beta_{11} n_1 + \beta_{12} n_2 + k_1 v, \tag{2.4a}$$

$$\delta_2' = \delta_2 + \beta_{21} n_1 + \beta_{22} n_2 + k_2 v, \tag{2.4b}$$

where β_{ij} are the light-shift coefficients (see Sec. III). k_1 (k_2) is the wave-number corresponding to mode 1 (2) and v the velocity of the center of mass of the atom.

Equation (2.3) is valid only if $\delta_1 \neq 0$ and $\delta_2 \neq 0$, which we suppose to be always the case (no resonant intermediate level).

Consider now the interference effect. The condition for perfect destructive interference $[\Gamma_g(n_1,n_2;v)=0]$ is

$$\delta_1' = -\frac{v_1 v_1'}{v_2 v_2'}\delta_2' = \gamma \delta_2' \quad \left(\gamma = -\frac{v_1 v_1'}{v_2 v_2'}\right). \tag{2.5}$$

The ionization probability thus vanishes if the following condition relating the system's parameters and degrees of freedom is satisfied:

$$\delta_1 - \gamma\delta_2 = (\gamma\beta_{21} - \beta_{11}) n_1 + (\gamma\beta_{22} - \beta_{12}) n_2 + (\gamma k_2 - k_1) v. \tag{2.6}$$

This equation is the key for understanding the generation of coherence by interference: Suppose one starts with a quantum superposition $\Sigma_{n_1,n_2,v} A(n_1,n_2;v)|g;n_1,n_2;v\rangle$ where v is an eigenvalue of the velocity operator corresponding to the eigenstate $|v\rangle$. After a time t such that $\Gamma_g(n_1,n_2;v)t \gg 1$ for all combinations of n_1, n_2, and v that do not satisfy Eq. (2.5), there will be a correlation between the ground state and the states $|n_1,n_2;v\rangle$ that satisfy Eq. (2.5), as all other states have a great probability to have made a transition into the continuum and are thus no longer correlated to the ground state. One thus sees that the quantum interference has generated a complicated kind of correlation among an internal (ground) state of the atom, the modes of the electromagnetic field, and an external atomic degree of freedom (the motion of the center of mass).

Naturally, this coherence is obtained at the price of a decrease in the probability of finding the atom at the ground state. The greater the degree of correlation, the lesser the probability of finding the atom in the ground state. This is an important limitation of this method.

Let us now outline two concrete examples of the generation of coherence by interference that will be studied in greater detail in the following sections: the generation of coherence among the ground state and the field modes (Sec. III) and between the ground state and the center-of-mass velocity (Sec. IV). In both cases, the underlying physical mechanism is exactly the same, and the only important difference is the relative weight of the light shift and the Doppler shift.

First suppose that the atom has been previously cooled to such a low temperature that Doppler effect is negligible compared to the light shifts. In this case, Eqs. (2.6) and (2.4) imply that the ground state will become correlated with photon-number states satisfying the relation

$$n_2 = \alpha n_1 + \mu \tag{2.7}$$

with

$$\alpha = \frac{\beta_{11} - \gamma\beta_{21}}{\gamma\beta_{22} - \beta_{12}}, \quad \mu = \frac{\delta_1 - \gamma\delta_2}{\gamma\beta_{22} - \beta_{12}}. \tag{2.8}$$

The second application corresponds to the opposite limit in which the light shift is negligible compared to the Doppler effect. If we take in this case the two modes of the electromagnetic field to be counterpropagative ($k_1 = -k_2 = k$), the kind of coherence that will be generated relates the atomic internal state to the atomic external degree of freedom corresponding to the motion of the center of mass. The ground state will be correlated with the velocity state having [cf. Eq. (2.6)]:

$$v = \frac{\gamma\delta_2 - \delta_1}{k(\gamma+1)}. \tag{2.9}$$

As will be shown in Sec. IV, this method can be used for the selection of a velocity class.

Let us now discuss the hypothesis we have made in neglecting the noninterfering couplings to the continuum. If the respective dipole matrix elements are v_i'' corresponding to the coupling between the intermediate state $|e_i\rangle$ and the continuum by absorption of a photon in mode i, the related ionization rates are

$$\Gamma_i = \frac{2\pi}{\hbar^2} f_0 n_i (n_i - 1)\left|\frac{v_i v_i''}{\hbar\delta_i'}\right|^2. \tag{2.10}$$

First note that these transitions do not prevent the quantum coherence involving the ground state to be built, as they induce irreversible transitions to the continuum (we neglect the small recombination probability that can exist in practical cases). If the atom is found to be in the ground state, this means *that it has not followed one of these paths into the continuum*. On the other hand, the method presented here is handicapped by the presence of these transitions, as the probability of finding the atom in the ground state after a given time will be smaller than if these couplings were absent. If we take a "real" atom with a $J=0$ (J being the total angular momentum) ground state coupled to $J=2$, $m=0,\pm 2$ levels in the continuum through a $J=1$, $m=\pm 1$ intermediate level [14], symmetry implies that $v_1' = v_2'$ and $v_1'' = v_2''$, and the Clebsch-Gordan coefficients corresponding to a $J=1 \rightarrow J=2$ transition imply that $v_1''/v_1' = \sqrt{6}$. The total transition ratio is thus (we neglect the corrections on the detunings in the following order-of-magnitude calculations)

$$\Gamma_g' = \Gamma_g + \Gamma_1 + \Gamma_2 = \frac{2\pi f_0}{\hbar^2}\left[n_1 n_2 \left| \frac{v_1 v_1'}{\hbar\delta_1} + \frac{v_2 v_2'}{\hbar\delta_2} \right|^2 + n_1^2 \left| \frac{v_1 v_1''}{\hbar\delta_1} \right|^2 + n_2^2 \left| \frac{v_2 v_2''}{\hbar\delta_2} \right|^2 \right], \tag{2.11}$$

where we supposed that the photon numbers are high enough to allows us to take $n_1 - 1 \approx n_1$, $n_2 - 1 \approx n_2$. The last equation can be rewritten, defining the quantity $\Gamma_{12} = (2\pi f_0/\hbar^2) n_1 n_2 |v_1 v_1'/\hbar\delta_1|^2$, as

$$\Gamma_g' = \Gamma_{12}\left[\left(1 + \frac{\delta_1}{\delta_2}\right)^2 + 6\frac{n_1}{n_2} + 6\frac{n_2}{n_1}\left(\frac{\delta_1}{\delta_2}\right)^2 \right], \tag{2.12}$$

where we used the relations between the different dipole matrix elements discussed above ($v_1 = v_2$, $v_1'' = \sqrt{6} v_1'$, etc.).

Note, however, that due to the interference, Γ_g can be much smaller than Γ_g'. The effect of the noninterfering couplings is thus to lessen the probability of finding the atom in the ground state by a factor of about (we take $|\delta_1| \approx |\delta_2|$)

$$\exp\left[-6\left(\frac{n_1}{n_2} + \frac{n_2}{n_1}\right)\Gamma_{12} t \right]. \tag{2.13}$$

This factor can be very important compared to the ionization due to the interfering paths, and can in certain cases forbid the practical application of the present method.

There is a more complicated system in which the noninterfering coupling is absent (see Fig. 2). In this system, the quantum interference shows up in the two-photon coupling between two discrete levels. It is thus possible to choose a two-photon transition between two $J=0$ levels via the $m=\pm 1$ magnetic sublevels of a $J=1$ intermediate level (in which case modes 1 and 2 are circularly polarized in opposite senses). The upper level is coupled to a continuum by the interaction with another mode of the electromagnetic field. The detailed dynamics of this system is somewhat complicated and will be studied elsewhere [15]. However, within certain conditions, one can show that this system is equivalent to the simplest one considered before. Intuitively it is easy to understand that if the transition rate Γ_s from the upper level to the continuum is very high compared to its coupling to the other levels, once the system arrives in the upper level it will in almost all cases make a transition into the continuum, and everything happens as if the upper level were a $J=0$ continuum. More rigorously, one can show, using the well-known Fano approach [16], that in the case of strong coupling Γ_s the upper level $|s\rangle$ "merges" into the continuum [17], and that the systems shown in Fig. 2 and in Fig. 1 have the same behavior, with the corresponding couplings related by $hf_0|v_i'|^2 = |V_i'|^2(\Gamma_s/\delta_i^2)(i=1,2)$. Moreover, the new system presents two interesting characteristics: the first one is that ionization can be freely turned "on" and "off" simply by putting on or off the additional laser that couples the upper level to the continuum. The second one is that the ionization of the upper level plays the effective role of a measurement of the atomic state, which can be important to "trace" over the atomic internal variable (see Sec. III).

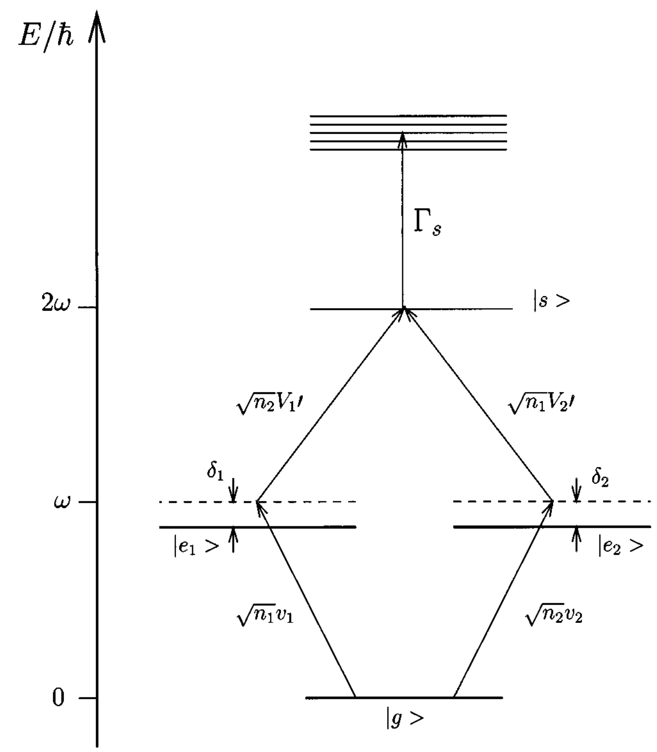

FIG. 2. The four-discrete-level model. The ground and the upper states are $J=0$ levels and the intermediate states are magnetic sublevels with $m=\pm 1$ of a $J=1$ level, the selection rules impose modes 1 and 2 to be $\sigma^{\pm}$ polarized, and the only possible couplings correspond to the absorption of one photon in each mode. For Γ_s strong enough, the upper level $|s\rangle$ will behave much like a continuum itself.

Having in mind this equivalence, we will only consider, in the rest of this paper, the simple model in which there is a direct two-photon coupling to the continuum and the noninterfering couplings are neglected (Fig. 1).

III. GENERATION OF COHERENCE INVOLVING THE FIELD MODES

In this section we study the generation of quantum coherence among the field modes and the ground state due to the combination of light shifts with quantum interference. This coherence, as we will see, can be transferred to the field by a measurement of the atomic state giving as a result the ground state. In order to simplify the formulas, we will study the simple case in which $\gamma = 1$[cf. Eq. (2.5)]. This hypothesis is not necessary, as it is possible to adjust the detunings in order to have destructive interference, and it plays no other role in what follows than simplifying formulas. We neglect the Doppler effect in the present section.

If $\alpha < 0$ the necessary and sufficient condition for the existence of nontrivial solutions of Eq. (2.7) is $\mu \geq |\alpha|$ (because n_1 and n_2 must be both positive), but then $n_1 \leq (\mu/|\alpha|)$ and $n_2 \leq \mu$. The case $\alpha < 0$ can thus be used to correlate the ground state with a *finite* superposition of number states in the two modes. Other particular cases that can be interesting for practical applications are as follows: If $\alpha \gg 1$ and $\mu = 0$, mode 2 becomes an "amplified replica" of mode 1. On the other hand, if $\alpha n_1 \ll \mu$ for all relevant values of n_1, the field in mode 2 will become very close to a number state $n_2 = \mu$.

Let us now precisely state the problem: Expressed in the base of number states, the field modes 1 and 2 can be written as

$$|F_1\rangle = \sum_{n_1=0}^{\infty} a_{n_1}|n_1\rangle, \quad |F_2\rangle = \sum_{n_2=0}^{\infty} b_{n_2}|n_2\rangle. \tag{3.1}$$

If the atom is initially in its ground state, the wave function of the global system at $t=0$ is

$$|\psi_0\rangle = |g\rangle \otimes |F_1\rangle \otimes |F_2\rangle = \sum_{n_1 n_2} a_{n_1} b_{n_2} |g;n_1,n_2\rangle \tag{3.2}$$

and the probability of finding, at $t=0$, n_1 photons in mode 1 and n_2 photons in mode 2 is thus

$$p_0(n_1,n_2) = |a_{n_1}|^2 |b_{n_2}|^2. \tag{3.3}$$

Because of the coupling with the continuum these probabilities are dumped out with a rate given by Eq. (2.3):

$$p(n_1,n_2;t) = p_0(n_1,n_2)\exp[-\Gamma_g(n_1,n_2)t]. \tag{3.4}$$

One can thus write the wave vector corresponding to long times as

$$|\Psi_\infty\rangle = |g\rangle \otimes \sum_{n_1} A(n_1,\alpha n_1+\mu)|n_1,\alpha n_1+\mu\rangle + f_0|s(0)\rangle \otimes \sum_{n_1,n_2 \neq \alpha n_1+\mu} A(n_1,n_2)|n_1,n_2\rangle \tag{3.5}$$

where the amplitude $A(n,m)$ is proportional to $a_n b_m$ within a phase factor. The last equation shows explicitly the kind of entanglement that has been generated among the field modes and the atomic state.

Let us put our main formulas in a more convenient form. Taking $\gamma=1$, the ionization rate is

$$\Gamma_g = \Gamma_0 n_1 n_2 \left| \frac{\omega}{\delta_1'} - \frac{\omega}{\delta_2'} \right|^2, \tag{3.6}$$

where we have introduced the quantity

$$\Gamma_0 = 2\pi f_0 \omega^2 \left| \frac{v_1}{\hbar\omega} \right|^2 \left| \frac{v_1'}{\hbar\omega} \right|^2. \tag{3.7}$$

The coefficients β_{ij} relating the photon numbers to the light shifts are obtained by simple application of second-order perturbation theory [13]. They read

$$\hbar\beta_{11} = 2\frac{|v_1|^2}{\hbar\delta_1}, \tag{3.8a}$$

$$\hbar\beta_{12} = \frac{|v_2|^2}{\hbar\delta_2} - \int d\delta f(\delta) \frac{|v_1'(\delta)|^2}{\hbar(\delta-\delta_1)}, \tag{3.8b}$$

$$\hbar\beta_{21} = \frac{|v_1|^2}{\hbar\delta_1} - \int d\delta f(\delta) \frac{|v_2'(\delta)|^2}{\hbar(\delta-\delta_2)}, \tag{3.8c}$$

$$\hbar\beta_{22} = 2\frac{|v_2|^2}{\hbar\delta_2}. \tag{3.8d}$$

The order of magnitude of the maximum time necessary for the building of the quantum coherence is $t_c \approx [\Gamma_0(\omega/\delta)^2]^{-1}$, with $\delta = \max(\delta_1,\delta_2)$. This time is an upper limit valid for all photon numbers in the range $[1,\infty)$. It is clear that if the photon numbers in a given situation are concentrated around given mean values, a more precise value for the correlation time will be $t_c' = t_c/(\langle n_1\rangle\langle n_2\rangle) \ll t_c$.

Suppose that we begin with two uncorrelated coherent states in modes 1 and 2, with initial mean photon numbers

$$\langle n_1\rangle = \nu_1, \quad \langle n_2\rangle = \nu_2 \tag{3.9}$$

and thus

$$p_0(n_1,n_2) = \exp[-(\nu_1+\nu_2)]\frac{\nu_1^{n_1}\nu_2^{n_2}}{n_1!\,n_2!}. \tag{3.10}$$

We take $\delta_1=\delta_2=\delta$, and thus $\mu=0$.

How can we describe the generation of the quantum coherence in an operational way? By defining the appropriate correlation function. Equation (3.5) shows that if the atom stays in the ground state, photon numbers are correlated: $n_2 = \alpha n_1$. This means that if we know that the atom is in the ground state (because we previously made a measurement of its state) and if a measurement of the photon number in mode 1 gives n_1, a subsequent measurement of the photon number in mode 2 will give αn_1 with no quantum uncertainty. Otherwise stated, the quantum fluctuations of the quantity $\delta n = n_2 - \alpha n_1$ vanish. We thus propose the following protocol: We wait for a time $t_f \gg t_c'$ and then make a measurement of the atom state. If the result is $|g\rangle$, we measure the quantum fluctuations $\langle \delta n^2\rangle - \langle \delta n\rangle^2$ of δn.

The generation of correlation — that is transferred to the field by the atomic state measurement performed at t_f — can then be translated into a decrease of the fluctuations of δn as t_f increases, which can be quantified by defining the quantity

$$R(\Gamma_0\omega^2 t_f/\delta^2) = 10\log_{10}\left(\frac{\langle \delta n^2(t_f)\rangle - \langle \delta n(t_f)\rangle^2}{\langle \delta n^2(0)\rangle - \langle \delta n(0)\rangle^2} \right), \tag{3.11}$$

where we have rescaled the time variable to the natural time unit of the system $\delta^2/(\Gamma_0\omega^2)$. The effect of the measurement of the atomic state is to "trace" over the internal atomic variable, transferring, due to the wave-packet reduction, the coherence into the electromagnetic field variables. Let us mention that the quantity R above is a straightforward generalization of the physical quantities experimentally measured in twin-photon experiments [5].

Figure 3 illustrates the dependence of R on t_f for different values of α and of the ratio $r = \nu_2/\nu_1$. The figure suggests that the best correlation is obtained with $r=\alpha$ and for the greatest initial mean photon numbers. A more rigorous argument allows us to generalize these observations:

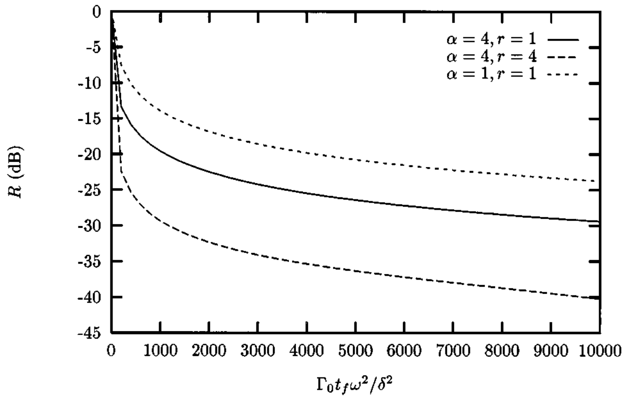

FIG. 3. Time evolution of the degree of correlation R. $r=\nu_2/\nu_1$, $\nu_1=100$, $\beta_{11}=\beta_{22}=10^{-4}\delta$, $\beta_{12}=\beta_{21}=2\times10^{-5}\delta$, as for the other figures of this section.

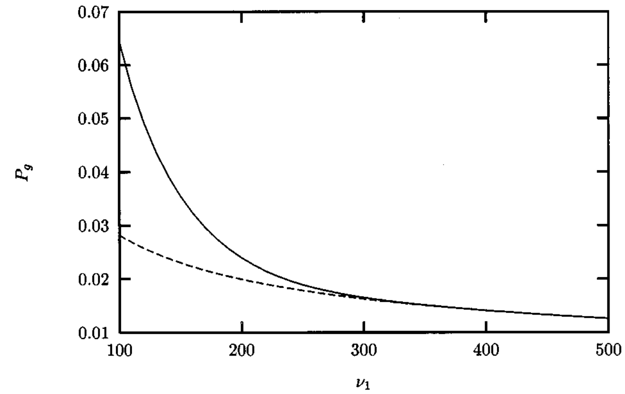

FIG. 5. Dependence of P_g on the initial photon number ν_1 for $\alpha=1$, $r=\alpha$, and $\Gamma_0 t_f(\omega/\delta)^2=10^4$. The solid curve is the "exact" numerical summation, whereas the dashed curve represents the a-symptotic expression obtained from Eqs. (3.12) and (3.15).

The increase of the correlation with the mean photon number comes from the following fact: Eq. (3.10) shows that there is a nonzero amplitude of finding, at $t=0$, zero photons (i.e., the vacuum) in one of the field modes. Obviously in this case no ionization process can take place. The "idle" states $|g;n_1,0\rangle$ or $|g;0,\ n_2\rangle$ remain correlated to the ground state, without being affected by the filtering action of the coupling with the continuum, violating the correlation law. There is thus a "contamination," in later times, of the "useful" ground-state correlated states that have been selected by the ionization process. It is clear that this contamination depends on the initial weight of the idle states. This "vacuum tail," for coherent states, decreases as the mean photon number in the corresponding mode increases, in accordance with what we observed in Fig. 3.

In order to have better evidence of the effect of the idle states we plotted in Fig. 4 the dependence of R (for a fixed time t_f) on the initial mean photon number ν_1 (with $\alpha=1$ and $\nu_2=\nu_1$). The correlation is greater if the initial photon number increases. On the other hand, Fig. 5 shows that this better correlation is again obtained at the price of a decrease of the ground-state population. This is not surprising, as a higher mean photon number means also a higher ionization rate.

The fact that the correlation is maximum for $\nu_2/\nu_1=\alpha$ (see Fig. 6) is a consequence of the filtering process: The population of the useful states at t_f is proportional to their initial population $p_0(n_1,\alpha n_1)$. If this population is small, the contamination by the idle states will have a larger weight. In order to maximize this population, one must choose an initial state in which the weight of the useful states is also maximum, namely, $\nu_2\approx\alpha\nu_1$.

In the particular case $\alpha=1$, an explicit calculation of these effects can be made. Let us evaluate the ground-state population at long times. Asymptotically ($t_f\to\infty$), only the useful and the idle states will remain correlated to the atomic ground state [see Eq. (3.5)]. Thus, the asymptotic value of the ground-state population will be given by

$$P_g=\sum_{n_1=0}^{\infty} p_0(n_1,n_1)+\sum_{n_1=1}^{\infty} p_0(n_1,0)+\sum_{n_2=1}^{\infty} p_0(0,n_2) \qquad (\alpha=1). \tag{3.12}$$

Consider first the useful states [first term on Eq. (3.12)]. This probability can be expressed, using Eq. (3.10), as

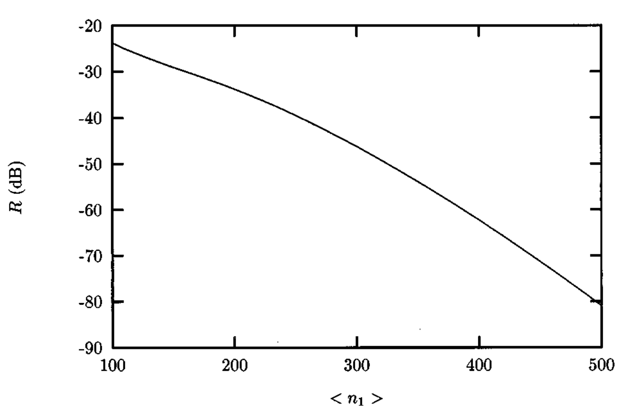

FIG. 4. Dependence of R on the initial photon number ν_1 for $\alpha=1, r=\alpha$, and $\Gamma_0 t_f(\omega/\delta)^2=10^4$.

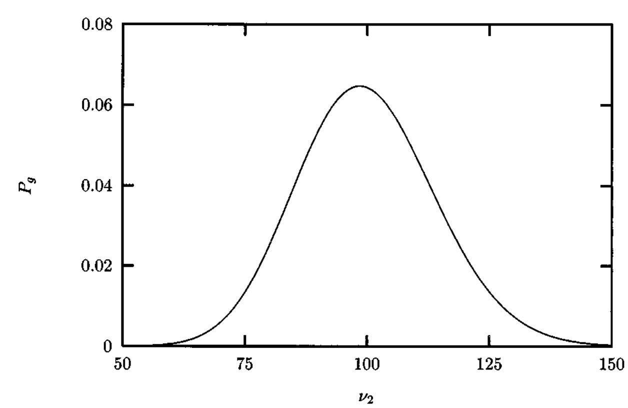

FIG. 6. Dependence of P_g on the initial mean photon number ν_2 for $\nu_1=100$, $\alpha=1$, and $\Gamma_0 t_f(\omega/\delta)^2=10^4$.

$$P_g^{(u)}=\exp[-(\nu_1+\nu_2)]\sum_{n=0}^{\infty}\frac{\nu_1^n\nu_2^n}{(n!)^2}$$
$$=\exp[-(\nu_1+\nu_2)]I_0(2\sqrt{\nu_1\nu_2})\quad(\alpha=1)\quad(3.13)$$

where $I_0(x)$ is the modified Bessel function of the first kind [18], and the superscript (u) stands for "useful."

If $2\sqrt{\nu_1\nu_2}\gg 1$ the asymptotic expression for $I_0(x)$ [18] can be used:

$$I_0(x)\rightarrow\frac{e^x}{\sqrt{2\pi x}}\quad(x\rightarrow\infty),\tag{3.14}$$

which produces for $P_g^{(u)}$ the asymptotic expression ($\nu_1\neq 0$, $\nu_2\neq 0$)

$$P_g^{(u)}\approx\frac{\exp[-(\sqrt{\nu_1}-\sqrt{\nu_2})^2]}{2\sqrt{\pi}(\nu_1\nu_2)^{1/4}}\quad(\alpha=1)\tag{3.15}$$

from which it follows that $P_g^{(u)}$ is maximum for $\nu_2\approx\nu_1$. In Fig. 5 the dashed line shows how the asymptotic curve corresponding to Eq. (3.15) converges to the exact result.

Another relevant quantity is the asymptotic value of the ratio between the populations of useful and idle states, which describes the degree of contamination of the coherence by the idle states. Using the asymptotic expression for $P_g^{(u)}$, this ratio is found to be

$$\frac{P_g^{(u)}}{P_g^{(\mathrm{idle})}}\approx\frac{1}{2\sqrt{\pi}(\nu_1\nu_2)^{1/4}}\frac{\exp(2\sqrt{\nu_1\nu_2})}{e^{\nu_1}+e^{\nu_2}}\quad(\alpha=1)\ .\tag{3.16}$$

Two limit cases will illustrate this effect. If $\nu_1=\nu_2=\nu$, the ratio of useful to idle population increases as $e^{\nu}/\sqrt{\nu}$, which confirms our expectation. On the other hand, if $\nu_1\gg\nu_2$, this ratio is found to be proportional to $(e^{-\nu_1}e^{\nu_2})/(\nu_1\nu_2)^{1/4}$, in accordance with our guess that the best initial condition is $\nu_1\approx\nu_2$ in the case $\alpha=1$.

Concluding this section, we can say that the system presented here has shown interesting properties in generating correlation involving two modes of the electromagnetic field. Unlike the usual method for generating "two-mode squeezed states" using optical parametric oscillators [4,5], that generate states with $\alpha=1$, $\mu=0$, our method is also able to generate more complicated types of quantum coherence.

IV. QUANTUM COHERENCE INVOLVING THE ATOMIC VELOCITY: SELECTION OF A VELOCITY CLASS

In this section we discuss the generation of quantum coherence involving the internal and external degrees of freedom of the atom and we neglect the light shifts compared to Doppler effect. We limit ourselves to the one-dimensional case (the generalization to the three-dimensional case being immediate). We keep the simplifying hypothesis of the previous section: $\gamma=1$ and $\delta_1=\delta_2=\delta$, and we neglect the noninterfering couplings.

Consider an atom moving with a nonrelativistic velocity v along the direction of propagation of the light field, and take the two field modes to be counterpropagative: $k_1=-k_2=k$ in Eqs. (2.4)

$$\delta_1'=\delta+kv,\tag{4.1a}$$

$$\delta_2'=\delta-kv.\tag{4.1b}$$

The photon numbers in modes 1 and 2 are supposed to be so high that the photon-number dispersion can be neglected. Within these assumptions, the transition ratio from the ground state to the continuum is

$$\Gamma_g(v)=\Gamma_0 n_1 n_2\left|\frac{\omega}{\delta+kv}-\frac{\omega}{\delta-kv}\right|^2.\tag{4.2}$$

Destructive quantum interference will thus exist for the atoms in the velocity class $v=0$.

We also suppose that the detunings are high enough that we can neglect the population of the intermediate levels and the repopulation of the ground state by spontaneous emission (this hypothesis is discussed below). This means that there is no coherence between the states $|g,v\rangle$ and $|g,v'\rangle$ $(v\neq v')$, and thus the evolution operator $U(t,t_0)$ is diagonal inside the space spanned by states of the form $|g,v\rangle$:

$$\langle g,v'|U(t,t_0)|g,v\rangle=\tilde{U}(t,t_0;v)\,\delta(v-v').\tag{4.3}$$

Up to the second order, $\tilde{U}$ is given by

$$\tilde{U}(t,t_0;v)\approx e^{-i\tilde{E}(v)t/\hbar}e^{-\Gamma_g(v)t/2}.\tag{4.4}$$

If the initial state is

$$|\phi(0)\rangle=\int dv'\,\phi_0(v')|g,v'\rangle\tag{4.5}$$

we find at time t,

$$\langle g,v|\phi(t)\rangle=\int dv'\,\phi_0(v')\langle g,v|U(t,0)|g,v'\rangle$$
$$=\phi_0(v)e^{-iE(v)t/\hbar}e^{-\Gamma_g(v)t/2}.\tag{4.6}$$

The velocity distribution at time t is thus

$$n(v,t)=|\langle g,v|\phi(t)\rangle|^2$$
$$=|\phi_0(v)|^2e^{-\Gamma_g(v)t}=n_0(v)e^{-\Gamma_g(v)t},\tag{4.7}$$

with $n_0(v)=|\phi_0(v)|^2$.

We have just shown that the problem can be stated in "classical" terms, and in what follows we will use the concept of velocity classes — more appropriate to practical applications — rather than the language of quantum amplitudes and probabilities. The former formulation is by far the most commonly used in the laser cooling literature [6].

The system is thus able to generate quantum coherence between the internal (ground) atomic state and a particular velocity class. In order to illustrate this effect, we plotted in Fig. 7 the velocity distribution of ground-state atoms obtained from Eq. (4.7) for different times beginning from a Maxwellian distribution of width δ/k. As the system evolves, this distribution becomes narrower and narrower,

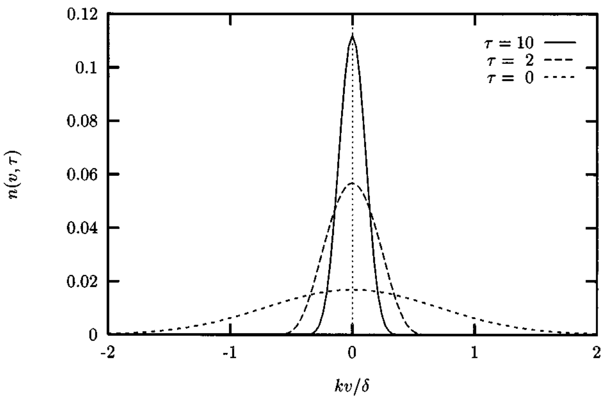

FIG. 7. Velocity distribution for ground-state atoms for different times. $\tau = \Gamma_0 n_1 n_2 t(\omega/\delta)^2$. The distribution for $\tau = 0$ is Maxwellian.

although keeping a quasi-Gaussian shape. In Fig. 8 we show the asymptotic behavior of the kinetic temperature T of the ground-state atoms, giving evidence of the ability of this system to perform the velocity selection.

For sufficiently low velocities such that $kv/\delta \ll 1$, which can be achieved, for example, by first cooling the atoms by a conventional method, an explicit analysis of the velocity selection process can be made. For that, we expand the ionization rate to the lowest order in v (namely, v^2):

$$\Gamma_g \approx g V^2 = 4\Gamma_0 n_1 n_2 \left(\frac{\omega}{\delta}\right)^2 \left(\frac{kv}{\delta}\right)^2, \tag{4.8}$$

where we introduced the normalized variable $V = kv/\delta$. If the initial velocity distribution is $n_0(V) = N_0 e^{-V^2}$, the velocity distribution at time t is [Eq. (4.7)]

$$n(V,t) = n_0(V)\exp[-gV^2 t] = N_0 \exp[-(1+gt)V^2]. \tag{4.9}$$

Taking into account that the kinetic temperature T is proportional to $\langle V^2 \rangle$, and that the total number of atoms at the instant t is given by

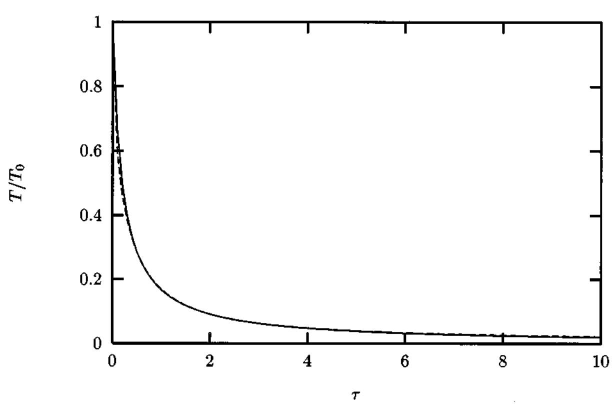

FIG. 8. Time evolution of the kinetic temperature of ground-state atoms. T_0 is the initial temperature. $\tau = \Gamma_0 n_1 n_2 t(\omega/\delta)^2$. The dashed line corresponds to the low velocity approximation given by Eq. (4.11)

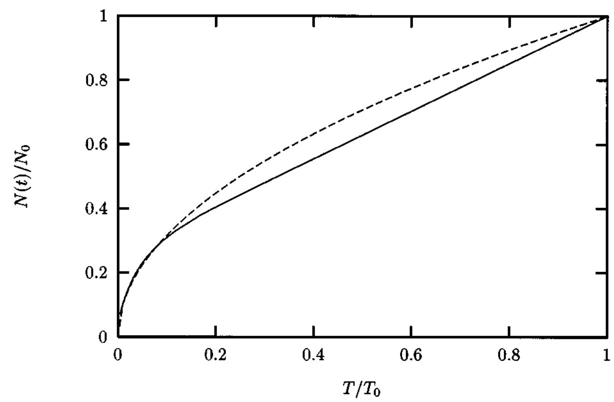

FIG. 9. Relation between the temperature and the survival probability of ground-state atoms. The dashed line corresponds to the low velocity approximation given by Eq. (4.13).

$$N(t) = \int_{-\infty}^{+\infty} n(V,t)\,dV \tag{4.10}$$

one finds after straightforward integrations

$$\frac{T}{T_0} = \frac{1}{1+gt}, \tag{4.11}$$

where T_0 is the initial temperature, and

$$\frac{N(t)}{N_0} = \frac{1}{\sqrt{1+gt}} \tag{4.12}$$

and thus

$$\frac{N(t)}{N_0} = \sqrt{\frac{T}{T_0}}. \tag{4.13}$$

These results are valid in the limit $V^3 \ll 1$.

In Figs. 8 and 9 the dashed lines show how these approximations compare to the exact results.

A detailed comparison of this method of velocity selection with the usual methods of laser cooling (Doppler [7], Sisyphus [8]) or velocity selection beyond the photon-recoil limit (coherent population trapping [9] or Raman [10]) is out of the scope of the present work and will be considered elsewhere [19]. Let us just outline a few interesting points:

(1) Note that the three discrete levels of our system (Fig. 1) have the same structure as those of a magneto-optical trap (MOT). The MOT and the velocity selection method proposed here are thus compatible, and they can be combined.

(2) In practical applications, there is no need for an explicit measurement of the atom internal state to transfer the coherence into the external degree of freedom, as the ions, not interacting with the light, escape the cooling region (this plays the role of an implicit atomic state measurement).

(3) The present method is in principle *not limited by the spontaneous emission* as the transitions to the continuum are irreversible. It is thus able to achieve velocity selection below the photon recoil limit. However, transitions from the ground state to one of the excited states followed by spontaneous emission are still possible, and will contribute to the warming of the system. A detailed study of this problem

shows that, due to this warming, there is a lower limit for the achievable temperature, for a given ionization rate [19].

V. CONCLUSION

A relatively simple quantum system has been shown to be able to generate quantum coherence among the various degrees of freedom of an atom by the interaction with the electromagnetic field.

Thanks to the light shift, the system can generate quantum coherence involving the ground state and two modes of the field, and this coherence can be transferred to the field by a measurement process, leading to an ''EPR-type'' correlated state, that can be used for example in sub-shot noise measurements.

The system has also shown ability to generate coherence between the ground state and one particular velocity class, allowing velocity class selection.

[1] A. Einstein, B. Podolski, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[2] J.S. Bell, Physics **1**, 195 (1964).

[3] R.E. Slusher, L.W. Hollberg, B. Yurke, J.C. Mertz, and J.F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).

[4] L. A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).

[5] A. Heidmann, R.J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, and G. Camy, Phys. Rev. Lett. **59**, 2555 (1987).

[6] See, e.g., J. Opt. Soc. Am. B **6**, (1989), special issue on laser cooling and trapping, edited by S. Chu and C. Wieman.

[7] T. W. Hänsch and A. Schawlow, Opt. Commun. **13**, 68 (1975).

[8] P. Lett, R. Wats, C. Westbrook, W. D. Phillips, P. Gould, and H. Metcalf, Phys. Rev. Lett. **61**, 169 (1988); Y. Shevy, D. S. Weiss, P. J. Ungar, and S. Chu, *ibid.* **62**, 1118 (1989); J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2023 (1989).

[9] A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, Phys. Rev. Lett. **61**, 826 (1988); J. Lawall, F. Bardou, B. Saubamea, K. Simizu, M. Leduc, A. Aspect, and C. Cohen-Tannoudji, Phys. Rev. Lett. **73**, 1915 (1994).

[10] M. Kasevich and S. Chu, Phys. Rev. Lett. **69**, 1741 (1992); N. Davidson, H. J. Lee, M. Kasevich, and S. Chu, *ibid.* **72**, 3158 (1994).

[11] J. C. Garreau, Opt. Commun. **110**, 228 (1994).

[12] M. Goppert-Meyer, Ann. Phys. **9**, 273 (1931); B. Cagnac, G. Grynberg, and F. Biraben, J. Phys. (Paris) **34**, 845 (1973).

[13] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Processus d'Interaction Entre Photons et Atomes* (InterEditions, Paris, 1988) [English translation: *Atom-Photon Interactions: Basic Process and Applications* (Wiley, New York 1992)].

[14] Naturally, there is also a coupling to the $J=0$ level of the continuum, but the relative strength of this coupling with respect to the coupling with the $J=2$ level cannot be deduced from pure symmetry arguments.

[15] J. C. Garreau (unpublished).

[16] U. Fano, Nuovo Cimento **12**, 156 (1935); Phys. Rev. **124**, 1866 (1961).

[17] The indirect coupling of a discrete level to a continuum through another strongly coupled discrete level is discussed in detail in Sec. III, Complement C_I of Ref. [13].

[18] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*, 9th ed. (Dover, New York, 1972).

[19] D. Wilkowski, J. C. Garreau, D. Hennequin, and V. Zehnlé (unpublished).