Self-pulsing instabilities in an optical parametric oscillator: Experimental observation and modeling of the mechanism

Pierre Suret, Dominique Derozier, Marc Lefranc, Jaouad Zemmouri, and Serge Bielawski
Laboratoire de Physique des Lasers, Atomes et Molécules, UMR CNRS, Centre d’Etudes et de Recherches Lasers et Applications, Université des Sciences et Technologies de Lille, F-59655 Villeneuve d’Ascq Cedex, France
(Received 16 February 1999; published 14 January 2000)

We have observed sustained self-pulsing in a continuously pumped, triply resonant, optical parametric oscillator. From the analysis of our experimental data, we conclude that the instability mechanism is different from the Hopf bifurcation predicted by the classical model of parametric interaction. Self-pulsing results from the interplay of a slow variable (temperature) and the optical bistability cycle, leading to a singularly perturbed system. From simple arguments, we propose a minimal dynamical model that reproduces well the observed behaviors.

PACS number(s): 42.65.Yj, 42.65.Sf

Continuous-wave optical parametric oscillators (OPOs) have recently received increased interest both experimentally and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically. Due to their tunability and to their quantum properties, OPOs are promising sources of coherent and theoretically.

It is thus important to understand the instabilities that cw OPOs may display, especially as they have been recognized as a system of choice for the study of complex nonlinear dynamics. In particular, theoretical studies have shown that OPOs are good candidates for the observation of various fundamental phenomena, such as transition to chaos [4], spatiotemporal dynamics [5], including localized structures and topological defects [6], as well as quantum images [7].

Yet, such a simple dynamical phenomenon as the self-pulsing instability observed in a triply resonant OPO [8] has not so far been given a clear interpretation. On the one hand, theoretical studies of the classical mean-field model of parametric interaction have predicted the existence of periodic behaviors originating in a Hopf bifurcation [4,9]. On the other hand, the first experimental observation of a self-pulsing regime suggested that a different mechanism might be involved [8]: transverse, thermal, and/or multimode effects possibly have to be taken into account. The nature of the self-pulsing instability thus remains an open and important question, especially as a good understanding of the temporal dynamics of OPOs is a first step towards investigating more sophisticated phenomena such as spatiotemporal dynamics.

In this paper, we identify a generic instability mechanism leading to self-pulsing behaviors, which differs from the Hopf bifurcation of the parametric model, and can result from several physical effects. The main ingredient is the interplay between the dynamics of a slow variable and the hysteretic cycle of a fast variable. Slow dynamics can be due to, e.g., thermal [10] or photorefractive effects [11], which are generally ignored in the modeling of OPOs.

Such instabilities are classical in perturbation theory [12,13]: adding a slow variable with a small relaxation rate $\epsilon$ is known to lead to a singularly perturbed system, which can display a dynamical instability not present in the $\epsilon=0$ case, even for small $\epsilon$. Examples of such instabilities are found in various fields, a classical one being the Van der Pol oscillator in the singular limit [12].

First we present the experimental observations that motivate the present work. We conclude (i) that the observed instability differs from the Hopf bifurcation predicted in Ref. [4] and (ii) that the model of the OPO must involve a variable that is much slower than the optical ones. In the second part, we propose and analyze a minimal model that reproduces the observed dynamical behaviors. We show that the peculiar shape of the regimes can be explained by a mechanism of periodic bursting, where the slow variable oscillates around the bistability cycle linked to the fast optical variables.

Our experiments have been carried out with a classical triply resonant OPO (i.e., resonant for the pump and parametrically generated signals) [8]. Because we are mainly interested in studying dynamical effects, the cavity length is not stabilized by a feedback loop. However, thanks to a careful design of the mechanical properties of the cavity and probably to thermal self-locking, as in Refs. [10,14], stationary or periodic regimes can be stably obtained for several minutes at fixed, uncontrolled, cavity length.

The experimental setup is displayed in Fig. 1. The nonlinear crystal is a 7-mm-long KTP crystal (Crystal Laser) cut for type-II phase matching. To avoid transverse effects, the cavity is kept far from the stability limit (even in the presence of thermal lensing) by choosing a length $L=5$ cm and

![FIG. 1. Experimental setup. Notation used: M$_1$, input mirror ($R_{\text{max}}$ at 1.06 $\mu$m, $R=90\%$ at 532 nm); M$_2$, output mirror ($R=99\%$ at 1.06 $\mu$m, $R_{\text{max}}$ at 532 nm); DM, dichroic mirror ($R_{\text{max}}$ at 532 nm and $T_{\text{max}}$ at 1.06 $\mu$m); PC, polarizing cube; BS, beam splitter; FP, Fabry-Perot spectrum analyzer; P, S, and I are the photodiodes monitoring the pump, signal, and idler intensities, respectively.](image-url)
mirrors with a curvature radius of 3 cm. Mirror coatings are such that the cavity is resonant at frequencies of the pump and of its first subharmonic (reflection coefficients are given in the caption of Fig. 1). Taking into account the vendor-specified absorption coefficients of the crystal (2.5% cm⁻¹ at 532 nm and 0.1% cm⁻¹ at 1064 nm), the cavity finesse is estimated to be 45 for the pump and 550 for the signal and idler fields. The pump is provided by a monomode frequency-doubled Nd:YVO₄ laser (Coherent Verdi, 5 W of maximum power). Threshold power is less than 25 mW, and either stable or self-pulsing operation can be obtained depending on the cavity length and pump power. In order to extract the core of the mechanism leading to self-pulsing, we focus here on regimes involving only a single longitudinal and transverse mode, as was checked with an external Fabry-Perot analyzer.

The output intensities at 532 nm and 1.064 μm are detected by silicon and InGaAs photodiodes, respectively. Because the time evolutions of the signal and idler fields appear to be identical, only the former will be represented here. However, as we will see below, monitoring the time evolution of the pump provides key information on the ingredients to include in the modeling of the OPO.

The observed self-pulsing regimes, such as the typical one displayed in Fig. 2, are found to differ significantly from those predicted from the purely parametric model: (i) time scales (∼10⁻⁴ s) are a few orders of magnitude slower than cavity lifetime (∼10⁻⁷ s); (ii) signal and pump display discontinuities, whereas smooth waveforms would be expected; (iii) during the periods of vanishing signal, the pump continues to evolve on a slow time scale, whereas it should remain constant after a short transient of a few microseconds.

These discrepancies clearly show that a more complete modeling is required. Fortunately, the last observation clearly reveals how to proceed. When the signal is off, the intracavity pump power should depend on the cavity detuning only, as the cavity then behaves as a mere Fabry-Perot interferometer. The unexpected slow evolution thus indicates that the effective cavity length is no longer a parameter, but is slaved to a new dynamical variable, which relaxes on a slow time scale. This slow variable can reflect changes in crystal temperature in the case here and in the experiments of Hansen and Buchhave [10], and probably in the experiments of Richy et al. [8], but can also be potentially due to a range of effects such as photorefractive changes in the index of refraction [11].

During self-oscillation, no mode hop occurs for the signal and idler fields. Due to the frequency-tuning properties of type-II OPOs [15], this gives an upper bound on the maximum variation ΔL of the cavity round-trip length. Indeed, one should observe mode hops in our configuration as soon as ΔL/λₚ=2×10⁻⁵, where λₚ is the pump wavelength (i.e., ΔL≈10 nm). That such a tiny effect induces a large variation in the output powers clearly calls for a dynamical interpretation.

In order to show that such a slow dynamics can indeed induce self-pulsing regimes, we consider the simplest OPO model [4,9] (mean-field approximation, degenerate case, and without transverse effects) coupled to a slow variable θ as follows: (i) cavity detunings are assumed to be functions of θ, as discussed above; (ii) the time evolution of θ is driven by pump and signal intensities, as in, e.g., thermal effects (this should, however, be a generic coupling for a scalar variable). Since the cavity is far from degeneracy and given the high cavity finesse, the effect of variations of the cavity geometry (such as induced by a change in the focal length of a thermal lens) is orders of magnitude below the effect of the path length change. Thus, we neglect the influence of θ on other model parameters, such as the input pump power.

If we set Ap and As the complex amplitudes of the pump and subharmonic, respectively, the corresponding normalized evolution equations read

\[ \dot{A}_p = \gamma [ E - (1+i\sigma_p(\theta))A_p - A_p^2 ], \]

\[ \dot{A}_s = - (1+i\sigma_s(\theta))A_s + A_p A_s^* , \]

\[ \dot{\theta} = \epsilon \phi(\theta, |A_p|^2, |A_s|^2), \]

where the time unit is the cavity decay time τ of the signal field (τ=4L/cT_s, with [L] the optical cavity length and T_s the transmission coefficient of the output mirror), γ is the cavity decay rate for the pump, E is the input pump, \( \sigma_p(\theta) \) and \( \sigma_s(\theta) \) are the normalized detunings associated with the pump and signal, respectively, and ε is the relaxation rate of the slow variable θ (ε≪1,γ).

We have seen above that θ should describe changes in the cavity effective length (such as those caused by the dependence of refraction indexes on temperature). For definiteness, we choose θ to be the variation of the signal detuning: \( \sigma_s(\theta) = \Delta_s - \theta \). To first order, the pump detuning is then given by \( \sigma_p(\theta) = \Delta_p - 2\theta/\gamma \) [16], and the simplest expression for ƒ is obtained by assuming that for fixed field intensities, θ relaxes exponentially to an equilibrium value depending linearly on these intensities:
where $\alpha$ and $\beta$ characterize pump and signal absorption by the crystal. Note that the feedback loop resulting from the coupling to $\theta$ naturally accounts for the self-locking observed in our experiments, and also reported in Ref. [10].

As expected, introduction of the singular perturbation (1c) leads to self-pulsing. What is more striking is that numerical integration of this model reproduces extremely well the waveforms observed experimentally. As an example, Fig. 3 shows a typical regime that closely matches the experimental one in Fig. 2 for similar values of the accessible parameters ($\gamma = 10$ and $E = 4$). Taking into account the normalization of the fields [4], the ratio $\gamma \alpha / \beta$ should be close to the ratio of the absorption coefficients of the crystal, and has thus been fixed at 25. The remaining free parameters $\alpha$ and $\epsilon$ have been chosen so that the correct time scale is obtained and the waveforms quantitatively reproduce those observed experimentally. With $\epsilon = 10^{-3}$, the thermal time scale is $\tau / \epsilon \approx 70 \ \mu s$, which gives the correct order of magnitude of the instability period (experimental and numerical periods are close to 90 $\mu s$).

To identify the mechanism and explain the particularities of the self-pulsing regimes, we take into account the large ratio of the two time scales, i.e., the smallness of $\epsilon$. This allows us to use our knowledge of the unperturbed system [4,9] for $\epsilon = 0$ to explain the properties of the actual system.

The variable $\theta$ is a parameter for the unperturbed system of Eqs. (1a) and (1b), whose stable stationary solutions will be noted $\bar{A}_p(\theta)$ and $\bar{A}_s(\theta)$. In first approximation, assume that in the complete model (1), the optical fast variables $(A_p, A_s)$ adiabatically follow the slow one $\theta$. The dynamics is then reduced to

$$f = - \theta + \alpha |A_p|^2 + \beta |A_s|^2,$$

where $\alpha$ and $\beta$ characterize pump and signal absorption by the crystal. Note that the feedback loop resulting from the coupling to $\theta$ naturally accounts for the self-locking observed in our experiments, and also reported in Ref. [10].

As expected, introduction of the singular perturbation (1c) leads to self-pulsing. What is more striking is that numerical integration of this model reproduces extremely well the waveforms observed experimentally. As an example, Fig. 3 shows a typical regime that closely matches the experimental one in Fig. 2 for similar values of the accessible parameters ($\gamma = 10$ and $E = 4$). Taking into account the normalization of the fields [4], the ratio $\gamma \alpha / \beta$ should be close to the ratio of the absorption coefficients of the crystal, and has thus been fixed at 25. The remaining free parameters $\alpha$ and $\epsilon$ have been chosen so that the correct time scale is obtained and the waveforms quantitatively reproduce those observed experimentally. With $\epsilon = 10^{-3}$, the thermal time scale is $\tau / \epsilon \approx 70 \ \mu s$, which gives the correct order of magnitude of the instability period (experimental and numerical periods are close to 90 $\mu s$).

To identify the mechanism and explain the particularities of the self-pulsing regimes, we take into account the large ratio of the two time scales, i.e., the smallness of $\epsilon$. This allows us to use our knowledge of the unperturbed system [4,9] for $\epsilon = 0$ to explain the properties of the actual system.

The variable $\theta$ is a parameter for the unperturbed system of Eqs. (1a) and (1b), whose stable stationary solutions will be noted $\bar{A}_p(\theta)$ and $\bar{A}_s(\theta)$. In first approximation, assume that in the complete model (1), the optical fast variables $(A_p, A_s)$ adiabatically follow the slow one $\theta$. The dynamics is then reduced to

$$f = - \theta + \alpha |A_p|^2 + \beta |A_s|^2,$$

where $\alpha$ and $\beta$ characterize pump and signal absorption by the crystal. Note that the feedback loop resulting from the coupling to $\theta$ naturally accounts for the self-locking observed in our experiments, and also reported in Ref. [10].

As expected, introduction of the singular perturbation (1c) leads to self-pulsing. What is more striking is that numerical integration of this model reproduces extremely well the waveforms observed experimentally. As an example, Fig. 3 shows a typical regime that closely matches the experimental one in Fig. 2 for similar values of the accessible parameters ($\gamma = 10$ and $E = 4$). Taking into account the normalization of the fields [4], the ratio $\gamma \alpha / \beta$ should be close to the ratio of the absorption coefficients of the crystal, and has thus been fixed at 25. The remaining free parameters $\alpha$ and $\epsilon$ have been chosen so that the correct time scale is obtained and the waveforms quantitatively reproduce those observed experimentally. With $\epsilon = 10^{-3}$, the thermal time scale is $\tau / \epsilon \approx 70 \ \mu s$, which gives the correct order of magnitude of the instability period (experimental and numerical periods are close to 90 $\mu s$).

To identify the mechanism and explain the particularities of the self-pulsing regimes, we take into account the large ratio of the two time scales, i.e., the smallness of $\epsilon$. This allows us to use our knowledge of the unperturbed system [4,9] for $\epsilon = 0$ to explain the properties of the actual system.

The variable $\theta$ is a parameter for the unperturbed system of Eqs. (1a) and (1b), whose stable stationary solutions will be noted $\bar{A}_p(\theta)$ and $\bar{A}_s(\theta)$. In first approximation, assume that in the complete model (1), the optical fast variables $(A_p, A_s)$ adiabatically follow the slow one $\theta$. The dynamics is then reduced to

$$f = - \theta + \alpha |A_p|^2 + \beta |A_s|^2,$$

where $\alpha$ and $\beta$ characterize pump and signal absorption by the crystal. Note that the feedback loop resulting from the coupling to $\theta$ naturally accounts for the self-locking observed in our experiments, and also reported in Ref. [10].

As expected, introduction of the singular perturbation (1c) leads to self-pulsing. What is more striking is that numerical integration of this model reproduces extremely well the waveforms observed experimentally. As an example, Fig. 3 shows a typical regime that closely matches the experimental one in Fig. 2 for similar values of the accessible parameters ($\gamma = 10$ and $E = 4$). Taking into account the normalization of the fields [4], the ratio $\gamma \alpha / \beta$ should be close to the ratio of the absorption coefficients of the crystal, and has thus been fixed at 25. The remaining free parameters $\alpha$ and $\epsilon$ have been chosen so that the correct time scale is obtained and the waveforms quantitatively reproduce those observed experimentally. With $\epsilon = 10^{-3}$, the thermal time scale is $\tau / \epsilon \approx 70 \ \mu s$, which gives the correct order of magnitude of the instability period (experimental and numerical periods are close to 90 $\mu s$).

To identify the mechanism and explain the particularities of the self-pulsing regimes, we take into account the large ratio of the two time scales, i.e., the smallness of $\epsilon$. This allows us to use our knowledge of the unperturbed system [4,9] for $\epsilon = 0$ to explain the properties of the actual system.

The variable $\theta$ is a parameter for the unperturbed system of Eqs. (1a) and (1b), whose stable stationary solutions will be noted $\bar{A}_p(\theta)$ and $\bar{A}_s(\theta)$. In first approximation, assume that in the complete model (1), the optical fast variables $(A_p, A_s)$ adiabatically follow the slow one $\theta$. The dynamics is then reduced to

$$f = - \theta + \alpha |A_p|^2 + \beta |A_s|^2,$$
ing conclusion that the crucial role in the instability is played by signal absorption, even if weaker than pump absorption. Indeed, we know that \[\tilde{A}^{\text{off}}(\theta) = |\tilde{A}^{\text{on}}(\theta)|\] at the bifurcation point \(L\) of the bistability cycle [Fig. 4(b)]. Hence, the derivatives of \(\theta\) at points \(L\) and \(U\) on the lower and upper branches verify \(f_U = f_L + \beta |\tilde{A}^{\text{on}}(\theta)|^2\). Since we must have \(f_L < 0\) and \(f_U > 0\), this clearly shows that \(\beta > 0\) is a necessary condition for the appearance of the instability. On the other hand, we have observed that self-pulsing regimes still occur if we set \(\alpha = 0\). Pump absorption, even if significant, thus cannot lead alone to the instability.

In conclusion, we have identified an instability mechanism for the OPO that differs from the Hopf bifurcation predicted by the parametric model. It is generic in that the main ingredients are (i) the bistability cycle known to exist in triply resonant OPOs and (ii) a slow variable whose existence can be due to several physical effects, the most common one being thermal variations of the crystal refraction indexes.

This result calls for further investigations in several directions. First, a complete bifurcation study of the model is needed, especially to determine which conditions the crystal absorption coefficients \(\alpha\) and \(\beta\) must verify for the instability to occur. Second, the straightforward extension of our model to operation on two longitudinal modes is promising as it reproduces well the bimode regimes that we have also observed experimentally. Last, but not least, it is important to reconsider the study of transverse dynamics in connection with the present mechanism. For instance, our preliminary experimental observation near the cavity stability limit suggests the existence of thermally induced transverse variations of refractive indices. To describe this phenomenon, the model should be extended by including transverse effects and a heat diffusion equation.

Note added. We recently became aware of a work by Douillet et al. describing an instability in their AgGaS\(_2\) OPO that seems very similar to that described in this work [21].

We thank J.-J. Zondy for communicating Ref. [21] prior to publication. We are most grateful to C. Fabre and A. Maître for stimulating discussions and very helpful advice. They introduced one of us (P.S.) to the experimental aspects of OPOs during a two-month stay at the Laboratoire Kastler-Brossel (Ecole Normale Supérieure and Université de Paris VI). This work was funded for the most part by the Centre d’Études et de Recherches Lasers et Applications. The Laboratoire de Physique des Lasers, Atomes et Molécules is Unité Mixte de Recherche du CNRS. The Centre d’Études et de Recherches Lasers et Applications is supported by the Ministère chargé de la Recherche, the Région Nord–Pas de Calais, and the Fonds Européen de Développement Économique des Régions.

[16] We take this form for simplicity. More generally, if the derivatives of the refraction indexes of pump and signal with respect to temperature are different, \(\sigma_p = \Delta_p - 2\eta h\gamma\) with \(\eta \neq 1\).