Quantum Scaling Laws in the Onset of Dynamical Delocalization

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We study the destruction of dynamical localization experimentally observed in an atomic realization of the kicked rotor by a deterministic Hamiltonian perturbation, with a temporal periodicity incommensurate with the principal driving. We show that the destruction is gradual, with well-defined scaling laws for the various classical and quantum parameters, in sharp contrast to predictions based on the analogy with Anderson localization.

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Quantum chaos is defined as the dynamical behavior of a *quantum* system whose *classical* limit is chaotic. This has triggered a large number of studies trying to relate classical properties to quantum properties, e.g., Lyapunov exponents to quantum fidelity [1], or to detect quantum stability in a quantum-chaotic system [2].

Quantum-chaotic dynamics manifests itself by characteristic behaviors in which quantum interference plays an important role, making the dynamics distinct from classical dynamics. An example that shall concern us particularly here is *dynamical localization* (DL) [3], observed in time-periodic systems. DL is the suppression of the classical chaotic diffusion by quantum interference due to long-range coherence in momentum space; it manifests itself after a typical "localization time" as an exponential localization of the averaged momentum distribution. Because the system is time periodic, one can use the Floquet theorem to build a basis of quasieigenstates (states that are left unchanged, except for a phase factor, under the temporal evolution over one period). This makes it possible to map the quasieigenstates of the time-periodic system on the true eigenstates of a quasirandom static one-dimensional system, which presents the nontrivial Anderson localization. Anderson (or strong) localization has been a major subject in physics in the last decades, with implications in several areas, beyond the primary field of solid state physics [4]. In this Letter, we show that studying the breakdown of dynamical localization also brings some new insight on the physics of Anderson localization. The latter is known to be strongly dependent on the number of freedoms, with marginal localization in dimension 2 and the coexistence of localized and delocalized states—depending on the parameters—in dimension 3. By playing with the temporal dependence of the Hamiltonian—for example by adding incommensurate frequencies to make a quasiperiodic Hamiltonian—it is possible to study temporal analogs of the Anderson model in various dimensions. What happens if one introduces progressively a second (incommensurate) frequency in the system, by increasing its amplitude from zero? As the system is now quasiperiodic with two frequencies, it is reasonable to assume that it can be mapped onto a two-dimensional Anderson model [5] which, for a small perturbation, is a quasi-1D model. One then expects localization to be preserved, at the cost of an increased localization length [6]. In the present work, we show experimentally that this is *not* the case: it is a "diffusive scenario" that takes place, where DL is continuously destroyed by the perturbation. We unravel the scaling laws governing the phenomenon.

We consider an atomic version of the kicked rotor, a paradigmatic system for studies of classical [8] and quantum chaos [9–12]. It consists in exposing laser-cooled atoms to short, periodic pulses of a far-detuned standing wave. Using this system, DL has been unambiguously observed [13]. The temporal periodicity is a key ingredient. Random fluctuations on the strengths of the successive kicks, have been experimentally shown to destroy DL [9], even for fluctuations not significantly affecting the classical diffusive behavior. Similarly, the introduction of a small amount of non-Hamiltonian evolution (spontaneous emission) [9] is enough to induce decoherence that reduce or kill quantum interference effects, thus restoring the classical dynamics.

In previous works, we have extended the study of the kicked rotor to the two-frequency quasiperiodic case by adding a second series of kicks: the laser-cooled atoms interact with a modulated standing wave of wave vector $\mathbf{k}_L = k_L \mathbf{x}$ forming two series of kicks at frequencies $f_1 = 1/T_1$ (primary series) and $f_2 = rf_1$ (secondary series), so as to obtain a system described by the Hamiltonian

$$H = \frac{P^2}{2} + \sin\theta\left[K\sum_{n=0}^{N-1}\delta(t-n) + aK\sum_{n=0}^{rN-1}\delta\left(t-\frac{n}{r}\right)\right], \tag{1}$$

where we measure momentum in units of $2\hbar k_L$, $\theta = 2k_L x$, time in units of T_1. The normalized kick amplitude is $K = \Omega_1^2 \hbar k_L^2 \tau T_1/(2M\Delta)$ (Ω_1 is the resonant Rabi frequency, proportional to the light intensity, Δ is the detuning of the laser with respect to the atomic resonance, and τ is the duration of the kicks. In such units, the normalized Planck constant, describing the ''quanticity'' of the system, is $\hbar\!\!\!/ = 4\hbar k_L^2 T_1/M$; it can thus be controlled by changing the frequency of the kicks. We have shown that, in the quasiperiodic case (r irrational), with $a = 1$, DL is apparently destroyed [11]. Because of the finite duration of the experiment, it is not known whether DL exists at very long time (in the Anderson scenario, the localization time increases exponentially with a) or is simply completely destroyed.

What are the scaling laws for the onset of delocalization? In order to understand the origin of such laws, we analyze pertubatively the effect of the second series. The effect of each individual kick is expressed by a unitary evolution operator:

$$U(a, K, \hbar\!\!\!/) = \exp\left(-i\frac{aK\sin\theta}{\hbar\!\!\!/}\right). \tag{2}$$

For sufficiently small a—such that $aK/\hbar\!\!\!/ \ll 1$—this operator is close to unity and a single kick only slightly modifies the atomic state. It is the accumulation of a series of small kicks which significantly perturbs the dynamics. If the ratio r of the two frequencies is sufficiently far from any simple rational number, the second series of kicks is applied at quasirandom phases (measured with respect to the principal sequence), so that there is no coherent action of consecutive kicks. In classical language, the positions θ at consecutive secondary kicks are uncorrelated. In the unperturbed Floquet basis, the *incoherent* cumulative effect of the secondary kicks results in a diffusive process. The strength of a single secondary kick being proportional to $aK/\hbar\!\!\!/$, the incoherent cumulative effect of n secondary kicks is proportional to $na^2K^2/\hbar\!\!\!/^2$, and the characteristic time scale for the effect of the secondary kick series then scales as $T_2\hbar\!\!\!/^2/(a^2K^2)$. The other important time scale in the problem is the localization time, scaling like $T_1K^2/\hbar\!\!\!/^2$. If the cumulated effect of the secondary kick sequence during the localization time is small enough, DL has time to establish before being destroyed. On the opposite, for strong enough secondary kicks, diffusion in the Floquet basis is dominant and no localization is expected. The crossover between the two regimes arrives when the two time scales are comparable, i.e., when $T_2\hbar\!\!\!/^2/(a^2K^2) \simeq T_1K^2/\hbar\!\!\!/^2$, or (assuming r is of the order of unity)

$$\tilde{a} \equiv \frac{aK^2}{\hbar\!\!\!/^2} \simeq 1. \tag{3}$$

One sees that $\tilde{a}$ represents the scaled parameter governing the onset of delocalization. It depends on both the ''chaoticity'' parameter K and the quanticity parameter, the effective Planck constant $\hbar\!\!\!/$, which shows the intrinsic quantum nature of the phenomenon. However, if the preceding discussion establishes the form of the relevant parameter $\tilde{a}$, it is not sufficient for deciding whether the Anderson or the diffusive scenario actually takes place.

The experimental setup has been described elsewhere [14]. Cesium atoms are trapped and cooled in a magneto-optical trap, down to a temperature around 3 μK. The trap is turned off, and the atoms interact with the doubly pulsed standing wave [Eq. (1)]. Raman stimulated transitions are used to measure the population $\Pi(P)$ of a given momentum class, which can be chosen by changing the Raman detuning. The degree of localization can be directly determined by measuring the population in the zero momentum class $\Pi_0 = \Pi(P = 0)$. As the number of atoms in a given run is constant, Π_0 is smaller if the momentum distribution is larger, that is $\Pi_0 \propto \langle\Delta P^2\rangle^{-1/2}$. We measure Pi_0 by counting the number of atoms around zero-momentum, in a range narrower than the width of the initial distribution (few recoil momenta) and much smaller than the final width. The standing wave has a one-way power ranging from 90 to 120 mW and its frequency is continuously monitored by a lambda meter.

In order to study the destruction of DL, we choose a number of kicks that is larger than the localization time $N_L \propto (K/\hbar\!\!\!/)^2$ and measure Π_0 for increasing values of a from zero to 0.25. Figure 1 displays the typical results for seven sets of parameters, that are shown in Table I. Visual inspection of Fig. 1 does not reveal any obvious scaling law relating the different curves.

Numerical simulations of the kicked rotor quantum dynamics are useful for a detailed interpretation of the results. A few complications must be included in our simulations, which are, ordered by decreasing importance, the finite temporal length of the pulses which makes the

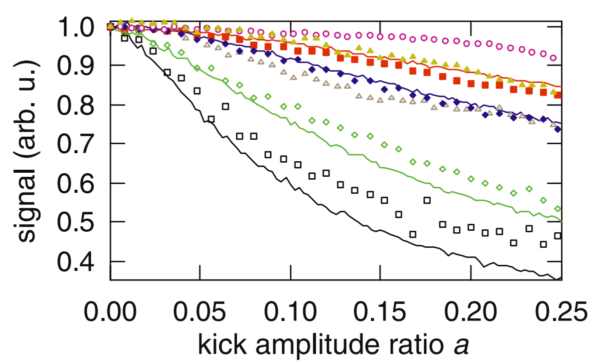

FIG. 1 (color online). Number of zero-velocity atoms as a function of the ratio a of amplitudes of the two series of kicks for various values of $\hbar\!\!\!/$ and K. In order to ease the comparison, the curves have been normalized so that the value at $a = 0$ is 1 for all curves. Parameters and plotting conventions are listed in Table I. Solid lines are numerical simulations for the curves at $K = 6.8$, with no adjustable parameter: the kick strength at the center of the atomic cloud is calculated from the properties of the beam, the temporal profile of the kicks is measured and spontaneous emission rates are calculated from the measured laser intensity, detuning, and from the oscillator strength of the atomic transition. Fluctuations on the solid lines are due to numerical noise.

TABLE I. Sets of parameters used in the curves of Fig. 1. The parameters $r = f_2/f_1 = 0.681$ and the ratio $N/N_L \approx 2.5$ (N_L is the localization time for $a = 0$) are the same for all data series. The pulse duration τ is 0.6 μs for the four top lines and 0.7 μs for the bottom three ones.

f_1 (kHz)	Δ (GHz)	P (mW)	N	N_L	K	$k\!\!\!^{-}$	Symbol
30.000	−18.8	95	35	14	6.8	3.46	■
36.000	−15.6	95	50	20	6.8	2.88	◆
54.000	−10.5	95	113	45	6.8	1.92	◇
72.000	−7.9	95	200	79	6.8	1.44	□
30.000	−21.3	62	18	7	4.5	3.46	○
30.000	−21.3	87	35	14	6.3	3.46	▲
30.000	−21.3	123	70	28	8.9	3.46	△

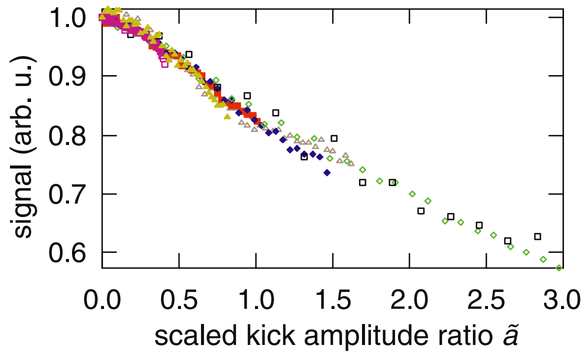

FIG. 2 (color online). Number of zero-velocity atoms as a function of the scaled kick series amplitude ratio, $\tilde{a} = aK^2/k\!\!\!^{-2}$. One observes the superposition of all curves. Parameters and plotting conventions are listed in Table I (as in Fig. 1).

kicks slightly different from δ-kicks, the spatial variation of the laser intensity across the atomic cloud (which implies that all atoms do not feel the same K value) and residual spontaneous emission. Altogether, they affect the shape of the curves in a limited way: the decay of Π_0 with a is slower by about 20%. Figure 1 shows a comparison of the numerical calculation for the four curves at $K = 6.8$ and various $k\!\!\!^{-}$ values with the corresponding experimental curves: the agreement is very good. There is no adjustable parameter, all the quantities have been either directly measured or calculated from measured quantities and first principles. Small deviations are observed for the lowest curve. This is probably an experimental artifact due to the long duration of the full kick sequence (2.8 ms, compared to 1.2 ms for the upper curve). The atoms are freely falling due to gravity, and, as they escape the central region of the Gaussian-profiled standing wave, they see a lower light intensity and the momentum diffusion is slowed down, and so is the delocalization.

In the various series, a constant ratio $N/N_L = 2.5$ (N_L is the localization time at $a = 0$) is kept in order to insure that all experiments correspond to the same "localization stage". The other parameters have been chosen to allow us to vary either $k\!\!\!^{-}$ or K keeping all other parameters constant. We choose $r = 0.681$, a typical "irrational" number, i.e., far from any simple rational, in order to avoid DL and sub-Fourier resonances [15] which are observed in a narrow range around rational numbers.

Data in Fig. 1 clearly demonstrate that the destruction of DL by a second series of kicks is gradual and certainly not a phase transition. In order to evidence the universality of the observed behavior, we show in Fig. 2 the same data plotted as a function of the scaled amplitude $\tilde{a}$, given by Eq. (3). The seven experimental curves now coincide, which proves that $\tilde{a}$ is the truly relevant parameter.

Although the preceding results are clear-cut proofs that the second series of kicks gradually reduce the localization of the system, this can result from two completely different mechanisms. (i) "Diffusive scenario": The second series destroys the DL and restores the diffusive behavior of the classical system, with a diffusion constant smoothly increasing from zero (for vanishing a), leading to a Gaussian distribution. (ii) "Anderson scenario": The localization (i.e., the exponential shape) is preserved, but with a localization length L that increases *exponentially* with $\tilde{a}$. Which of these two scenarios is at play in our experiment can be decided by measuring the momentum distributions. The top plot at Fig. 3 shows the experimentally observed momentum distribution for various values of $k\!\!\!^{-}$ and $K = 6.8$. It shows a gradual evolution from an exponential shape (strictly observed for $\tilde{a} = 0$) to a Gaussian shape at $\tilde{a} = 5.6$ (in the diffusive regime). Moreover, the bottom plot of Fig. 3 shows that, even if the exponential shape in the wings is preserved when $\tilde{a}$ is varied from 0 to ~1, the localization length varies very little (we have also checked numerically that it does *not* increase exponentially with $\tilde{a}$), whereas the effect of the second series of (small) kicks is visible near the center of the distribution, which is broader and lower. We can thus conclude that the diffusive scenario (i) is the one at play in our experiment. This is interesting and intriguing, as previous theoretical works [5,7] on a slightly different quasiperiodic system—a single series of kicks amplitude modulated periodically at an incommensurate frequency—showed the Anderson scenario at play. We thus conclude that quasiperiodicity with two incommensurate frequencies in the driving of the system is not enough to determine whether the system is localized or not. In other words, quasiperiodic driving of a Hamiltonian system might not lead to a universal behavior. This is a rather difficult theoretical problem, never treated in the literature, to the best of our knowledge. Experiments on the quasiperiodically driven atomic rotor thus help to clarify this stimulating issue.

In conclusion, we have observed that the destruction of dynamical localization by the addition of a small Hamiltonian periodic perturbation of incommensurate frequency, leads to a gradual destruction of the localization, through a continuous growth of a residual diffusion constant, and *not* to the equivalent of Anderson localization in a 2D system.

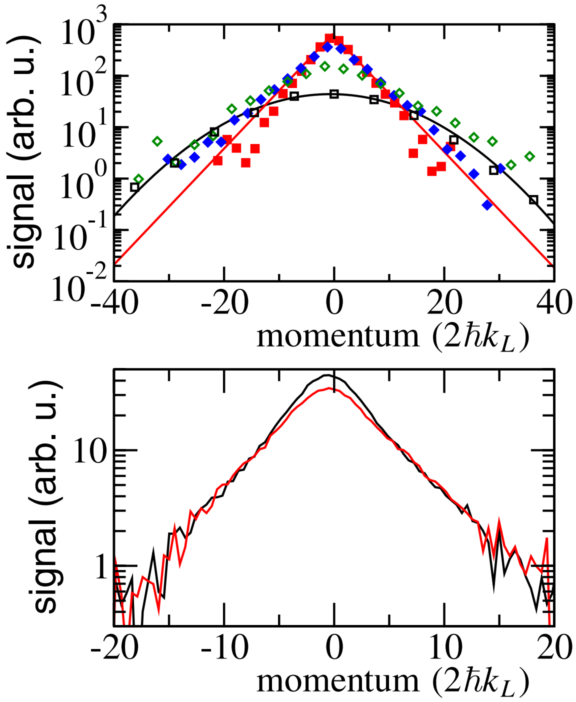

FIG. 3 (color online). *Top plot*: Experimental momentum distributions (in log scale) for fixed $a = 0.25$ and $K = 6.8$ and various values of $\kbar$. It shows the gradual evolution from an exponential shape at small $\tilde{a}$ to a Gaussian shape—characteristic of a diffusive behavior—at large $\tilde{a}$. *Bottom plot*: Momentum distributions (log scale) for fixed $K = 6.3$ and $\kbar = 3.46$ and (a) $\tilde{a} = 0$ and (b) $\tilde{a} = 0.83$. They coincide in the wings, showing that there is no significant increase of the localization length with $\tilde{a}$. Near the maximum at $P = 0$, the distribution for $\tilde{a} = 0.83$ is broader and lower, proving that the perturbation has destroyed DL and initiated a diffusive behavior (see parameters in Table I).

We have also determined and tested the quantum scaling laws governing the onset of delocalization.

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